SAT MATH REVIEW

The concepts covered on the SAT math sections include arithmetic and number theory, algebra and functions, geometry, and data analysis. You will not need to know matrices, logarithms, formal trigonometry, radians, standard deviation, or calculus.

I. Numbers and Arithmetic

- ***Properties of integers***
- ***Factors and multiples***
- ***Fractions, ratios and proportions***
- ***Percents***
- ***Exponents***
- ***Absolute value***
- ***Sets and sequences***

Integers are whole numbers, negative and positive. **Zero** is an integer, but is neither positive nor negative. **Rational** numbers are numbers that can be expressed as fractions. **Real** numbers include integers, rational numbers, and irrational numbers such as π, *e*, and √2.

Consecutive integers are integers in a sequence, each being 1 more than the previous integer:

$$n\,, (n\ +\ 1), (n\ +\ 2)\ \ldots$$

- If the sum of three consecutive integers is greater than 84, what is the smallest possible value of the first integer?

$$n\ +\ (n\ +\ 1)\ +\ (n\ +\ 2) > 84$$
$$3n + 3 > 84$$
$$n > 27$$

The smallest possible value of *n* is 28.

The **sum of** the consecutive integers from 1 to an integer *n* is equal to

$$(n + 1)\ \times {}^{n}/_{2}$$

- Take the set of consecutive integers from 1 to 30. The sum of the first and last integers is 1 + 30 = 31, the sum of the second and second from last integers is 2 + 29 = 31, and the sum of the third and third from last integers is 3 + 28 = 31. There will be 30/2 pairs of integers that each add to 31, so the total sum will be 31 × 30/2 = 465.

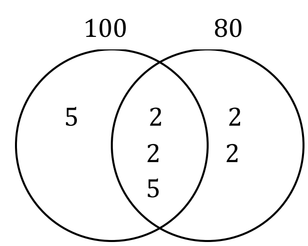
Adding or multiplying **even or odd integers** will result in a sum or product that is predictably even or odd:

- even + even = even
- odd + odd = even
- odd + even = odd
- even × even = even
- odd × odd = odd
- odd × even = even

An integer can be divided into a limited set of **factors**; there are an infinite number of **multiples** that can be divided by a specific integer. An integer with only 2 factors (itself and 1) is a **prime number.** 1 is not a prime number, but 2 is.

The **least common multiple** of two integers is the smallest positive integer that is a multiple of both integers. The **greatest common factor** of two integers is the largest integer that is a factor of both integers. You can determine both of these by finding the **prime factors** of each integer.

- What is the difference between the least common multiple and the greatest common factor of 100 and 80?

  Prime factors of 100: 2 × 2 × 5 × 5

  Prime factors of 80: 2 × 2 × 2 × 2 × 5

  100 and 80 share two 2s and one 5. Multiplying these prime factors together gives us their greatest common factor: 2 × 2 × 5 = 20.

  In total, four 2s and two 5s appear in these prime factorizations, excluding repeats. Multiplying these prime factors together gives us their least common multiple: 2 × 2 × 2 × 2 × 5 × 5 = 400.

  The difference between the LCM and the GCF is 400 – 20 = 380.

A **fraction** generally expresses a part/whole relationship between two quantities. A **ratio** generally expresses a part/part relationship among two or more quantities.

- If the ratio of boys to girls in the class is 12 to 17 = 12:17 = $\frac{12}{17}$, then the fraction of boys in the entire class is $\frac{12}{12+17} = \frac{12}{29}$.

- If the ratio of green to blue to red marbles in the jar is 5:4:3, then the fraction of green marbles in the jar is $\frac{5}{5+4+3} = \frac{5}{12}$.

A **proportion** is an equation where two ratios are set equal to each other. Solve proportions by **cross-multiplying**.

$$\text{If } \frac{a}{b} = \frac{c}{d}\text{, then } ad = bc.$$

- A train has travelled 40 miles in the past 45 minutes. If the train is moving at a constant speed, what distance will it have travelled after 4 hours?

  Convert to the same units: 45 minutes = 0.75 hours.
  Set up a proportion:

$$\frac{40 \text{ miles}}{0.75 \text{ hours}} = \frac{x \text{ miles}}{4 \text{ hours}}$$

  Cross-multiply and solve:

$$40 \times 4 = 0.75x$$
$$0.75x = 160$$
$$x = 213\frac{1}{3} \text{ miles}$$

A **percent** can be represented as a fraction whose denominator is 100 or as a decimal: 75% = 75/100 = 0.75. Solve a percent problem by converting the percent into a fraction and cross-multiplying:

- 30 is 60% of what number?

$$\frac{30}{x} = \frac{60}{100}$$
$$3000 = 60x\text{, so } x = 50$$

Alternatively, you can set up an equation and convert the percent into a decimal. The word *of* means multiplication:

- 30 is what percent of 150? $30 = x \times 150$, so $x = 0.2 = 20\%$
- 30 is 60% of what number? $30 = 0.6x$, so $x = 50$

Percent increase or **percent decrease** is equal to

$$\frac{\textit{change in amount}}{\textit{original amount}} \times 100\%$$

- The price of apples has decreased from \$0.99/lb to \$0.79/lb. What is the percent decrease in price?

$$\frac{\textit{change in amount}}{\textit{original amount}} \times 100\% = \frac{\$0.99 - \$0.79}{\$0.99} \times 100\% = 20.2\%$$

- A book has been discounted 15% and its current price is \$12. What was its original price?

$$\frac{x - \$12}{x} \times 100\% = 15\%$$
$$x - \$12 = 0.15x$$
$$0.85x = \$12$$
$$x = \$14.12$$

Exponent notation indicates that a number is being multiplied by itself. The number is the **base** and the number of times it is being multiplied is the **exponent**. Exponents follow standard rules:

$a^1 = a$	$4^1 = 4$
$a^0 = 1$	$4^0 = 1$
$a^{-m} = \frac{1}{a^m}$	$4^{-2} = \frac{1}{4^2}$
$a^m a^n = a^{m+n}$	$4^2 \times 4^3 = 4^5$
$\frac{a^m}{a^n} = a^{m-n}$	$\frac{4^5}{4^2} = 4^3$
$(a^m)^n = a^{mn}$	$(4^2)^3 = 4^6$
$a^m b^m = (ab)^m$	$4^2 \times 3^2 = (4 \times 3)^2$
$\frac{a^m}{b^m} = (\frac{a}{b})^m$	$\frac{4^2}{3^2} = (\frac{4}{3})^2$
$a^{\frac{m}{n}} = \sqrt[n]{a^m}$	$4^{\frac{4}{5}} = \sqrt[5]{4^4}$

- If $5^{\frac{x}{2}} = \sqrt{5^a \times 5^b}$, then what is x in terms of a and b?

$$5^{\frac{x}{2}} = \sqrt{5^x}$$
$$\sqrt{5^a \times 5^b} = \sqrt{5^{a+b}}$$
$$\sqrt{5^x} = \sqrt{5^{a+b}}$$
$$x = a + b$$

The **absolute value** of a number is its distance away from 0 on a number line. The absolute value will always be positive.

$$\text{If} |x| = 4, \text{then } x = 4 \text{ } or \text{ } x = -4$$
$$\text{If} |x| < 4, \text{then } -4 < x < 4$$
$$\text{If} |x| > 4, \text{then } x < -4 \text{ } or \text{ } x > 4$$

- Solve: $$|x - 5| = 4$$ $$x - 5 = 4 \text{ or } x - 5 = -4$$ $$x = 9 \text{ or } x = 1$$
  This means x is 4 units away from 5 in either the positive or negative direction.

- A manufacturer of cereal will discard all boxes weighing less than 28.5 oz and more than 31.5 oz. What absolute value equation represents all weights x that will be discarded?

  28.5 oz and 31.5 oz are both 1.5 oz away from 30 oz. If these were the weights that were discarded, we would write
  $$|x - 30| = 1.5$$
  Test: $$x - 30 = 1.5 \text{ or } x - 30 = -1.5$$ $$x = 31.5 \text{ or } x = 28.5$$

  However, the manufacturer will discard all boxes that weigh *less* than 28.5 oz and *more* than 31.5 oz—that is, those that differ *more* than 1.5 oz from the standard weight of 30 oz. Our equation should read
  $$|x - 30| > 1.5$$
  Test: $$x - 30 > 1.5 \text{ or } x - 30 < -1.5$$ $$x > 31.5 \text{ or } x < 28.5$$

A **set** is an **unordered** collection of items. These can be numbers, colors, letters, days of the week, other sets, etc. The **union** of two sets is a set consisting of all of the elements of both sets. The **intersection** of two sets is a set consisting of only the shared elements.

- If A is the set of the first three positive even integers and set B = {1, 2, 3, 4}, what is the union of the two sets? What is the intersection of the two sets?

  A = {2, 4, 6} and B = {1, 2, 3, 4}, so their union is {1, 2, 3, 4, 6}. Their intersection is {2, 4}.

A **sequence** is an **ordered** list of numbers, often following a specific pattern. A sequence can be definite or indefinite. In an **arithmetic sequence**, the next term is created by *adding* a constant to the previous term. In a **geometric sequence**, the next term is created by *multiplying* a constant to the previous term.

- Arithmetic sequence: 2, 6, 10, 14, 18, 22 ...
- Geometric sequence: 3, 12, 48, 192, 768 ...

You may need to find the sum or average of certain terms in a sequence, or the value of a specific term in a sequence. To answer any sequence question, always write out at least 5 terms to establish the pattern. You will never be asked to derive the formula for a sequence, but you may need to do something of the sort in order to answer the questions. Keep in mind:

- If a_1 is the first term of an arithmetic sequence, *d* is the common difference between terms, and *n* is the number of terms, then
$$a_n = a_1 + d(n - 1)$$

- If a_1 is the first term of a geometric sequence, *r* is the common ratio between terms, and *n* is the number of terms, then
$$a_n = a_1(r)^{n-1}$$

- Each term in a sequence is 3 times the preceding term. If the first term is 2, what is the average of the 5th and 7th terms?
$$a_7 = 2 \times 3^6 = 1458$$
$$a_5 = 2 \times 3^4 = 162$$
The average of the two terms is
$$\frac{1458 + 162}{2} = 810$$

Some sequences are neither arithmetic nor geometric. Establish a pattern, but don't waste your time trying to find an algebraic formula for these!

- 1, 4, 5, 9, 14, 23, 37: each term is the sum of the previous two terms.
- 4, 6, 9, 3, 4, 6, 9, 3: the terms repeat cyclically.

- A lane divider in a swimming pool has 77 flags strung in a row. The colours repeat in a pattern: red, blue, green, red, blue, green, red, blue, green ... If the first flag is red, what colour is the last flag?

  The three colours will repeat themselves 77/3 = 25.66667 times, or 25 times with two remainders. Following the pattern, these last two flags will be red and blue, respectively. Thus, the last (77th) flag is blue.

II. Algebra and Functions

- ***Operations involving polynomials***
- ***Equations and systems of equations***
- ***Properties of inequalities***
- ***Word problems***
- ***Functions and their domains, ranges***
- ***Strange symbols***
- ***Linear and quadratic graphs***
- ***Graph behaviour and translations***

Polynomials are algebraic expressions involving numbers and variables. You should already know how to factor and multiply polynomials. Keep in mind the **difference of squares** formula:

$$a^2 - b^2 = (a + b)(a - b)$$

- If $x^2 - y^2 = 25.6$ and $x + y = 8$, what is the value of $2x - 2y$?

$$x^2 - y^2 = (x + y)(x - y) = 25.6$$
$$8(x - y) = 25.6$$
$$x - y = 3.2$$
$$2x - 2y = 2(x - y) = 2 \times 3.2 = 6.4$$

  Remember what the question is asking—for this problem, it would have been a waste of time to solve for *x* and *y* individually!

You may need to solve single **equations** and **systems of equations**. To solve a system of equations, you can isolate a variable in one equation and substitute the resulting expression into the other equation. Sometimes you can also save time by adding or subtracting two or more equations.

- If 4 times a number minus a second number is equal to 1, and the sum of the two numbers is 9, what is their product?

  Create a system of equations:

$$4x - y = 1$$
$$x + y = 9$$

  Add: $5x = 10$

  Solve:

$$x = 2$$
$$y = 9 - 2 = 7$$
$$xy = 2 \times 7 = 14$$

Work with **inequalities** and **systems of inequalities** the same way you would work with equations and systems of equations, but keep in mind several key properties. Any operation involving positive numbers has no effect on the inequality, but multiplying or dividing by a negative number **reverses** the inequality. If both sides of an inequality are positive or negative, taking their reciprocal also **reverses** the inequality.

$$\text{If } a < b, \text{then } -a > -b$$
$$\text{If } a < b \text{ and } c \text{ is negative, then } ac > bc \text{ and } \frac{a}{c} > \frac{b}{c}$$

$$\text{If } a < b \text{ and both } a \text{ and } b \text{ are the same sign, then } \frac{1}{a} > \frac{1}{b}$$

- If $x^2 + y < 5$ and $y > 3$, give one possible value for x.

$$x^2 + y < 5, \text{ so } y < 5 - x^2$$
$$y > 3, \text{ so } 3 < y < 5 - x^2$$
$$3 < 5 - x^2$$
$$-2 < -x^2$$
$$2 > x^2$$
$$-\sqrt{2} < x < \sqrt{2}$$

$$x = 1, \frac{1}{2}, -\frac{1}{2}, -1 \ldots$$

You will frequently need to convert **word problems** into algebraic equations. Look for common words and phrases that correspond to mathematical operations:

Word/Phrase	**Translation**	**Symbol**
is, was, has, will be	equals	=
more, total, increased by, exceeds, gained, older, farther, greater, sum	addition	+
less, decreased, lost, younger, fewer, difference	subtraction	-
of, product, each	multiplication	×
for, per, out of, quotient	division	÷
at least	inequality	≥
at most	inequality	≤

- The total of Jake's and Amy's ages is 17. Last year, Amy was twice Jake's age. How old is Jake now?

  Key words: *total*, *last year,* and *twice*
  Let j represent Jake's current age and a represent Amy's current age:

$$j + a = 17$$

To represent the relationship between their ages *last year*, subtract 1 from each of their current ages:

$$a - 1 = 2(j - 1)$$

Solve for j:

$$a = 17 - j$$
$$17 - j - 1 = 2(j - 1)$$
$$16 - j = 2j - 2$$
$$18 = 3j$$
$$j = 6$$

Jake is currently 6 years old.

- An event has two admission prices: adult tickets are $10 each and student/senior tickets are $8 each. The ticket office has sold 77 tickets totalling $686. How many adult tickets were sold?

  Key words: *each*, *totalling*
  Let a represent the number of adult tickets and s represent the number of student/senior tickets:

  $$a + s = 77$$

  To represent total revenue, multiply each type of ticket by its price:

  $$10a + 8s = 686$$

  Solve for a:

  $$s = 77 - a$$
  $$10a + 8(77 - a) = 686$$
  $$10a + 616 - 8a = 686$$
  $$2a = 70$$
  $$a = 35$$

  There were 35 adult tickets sold.

You can often solve a word problem involving **rates** with the following equation:

$$distance = rate \times time$$

- An airplane flew from Mississippi to California. On the return trip from California to Mississippi, its average speed was reduced by 50 miles per hour and the resulting flight was $\frac{1}{10}$ longer than the original. What is the airplane's original average speed from Mississippi to California?

  Let d be the distance, r be the airplane's original speed and t be the time, in hours, of its flight from Mississippi to California.

$$d = r \times t$$

For its return trip, over the same distance, its speed is 50mph less and its time is $\frac{1}{10}$ greater:

$$d = (r - 50) \times \frac{11}{10} t$$

Solve for *r*:

$$r \times t = (r - 50) \times \frac{11}{10} t$$

$$r = (r - 50) \times \frac{11}{10}$$

$$r = \frac{11}{10} r - 55$$

$$\frac{1}{10} r = 55$$

$$r = 550$$

The plane's original speed was 550 mph.

- Adam can paint a house in 6 hours. When Karen comes to work, the two together can paint a house in 4 hours. Assuming that both work at a constant rate alone and together, how long does it take Karen to paint a house alone?

  We will use the same equation, but the distance *d* will represent the number of houses painted.

  $$d = r \times t$$

  Let Adam's rate be r_a. He paints 1 house in 6 hours:

  $$1 = r_a \times 6$$

  $$r_a = \frac{1}{6}$$

  Let Karen's rate be r_k. Their rate together is $(r_a + r_k)$. Together, they paint the same house in 4 hours:

  $$1 = (r_a + r_k) \times 4$$

  As Adam's rate is $\frac{1}{6}$, we can solve for r_k:

  $$1 = (\frac{1}{6} + r_k) \times 4$$

  $$\frac{1}{6} + r_k = \frac{1}{4}$$

  $$r_k = \frac{1}{12}$$

  Karen's rate is $\frac{1}{12}$ houses per hour, so she can paint 1 house in 12 hours.

A **function** is a rule that associates one set of numbers with another dependent set of numbers. The function $f(x) = x + 1$ tells us that every real number x is assigned to the corresponding number $x + 1$. If you are asked to find $f(3)$, simply follow the rule connecting 3 to its assigned number: $f(3) = 3 + 1 = 4$.

- If $(x) = 3x^2 - 2x$, what is $f\left(\frac{1}{2}\right) + f(1)$?

$$f\left(\frac{1}{2}\right) = 3\left(\frac{1}{2}\right)^2 - 2\left(\frac{1}{2}\right) = \frac{3}{4} - 1 = -\frac{1}{4}$$
$$f(1) = 3(1)^2 - 2(1) = 3 - 2 = 1$$
$$f\left(\frac{1}{2}\right) + f(1) = -\frac{1}{4} + 1 = \frac{3}{4}$$

- If $f(x) = 3x^2 - 2x$, what is $f(x^2)$?

$$f(x^2) = 3(x^2)^2 - 2(x^2) = 3x^4 - 2x^2$$

- If $f(x) = 3x^2 - 2x$, what is $f(f(2))$?

$$f(2) = 3(2)^2 - 2(2) = 12 - 4 = 8$$
$$f(8) = 3(8)^2 - 2(8) = 192 - 16 = 176$$
$$f\big(f(2)\big) = 176$$

The **domain** of a function is the set of all the numbers for which the function is defined—that is, all the "input" numbers for which the function still works. The **range** of a function is the set of all the "output" numbers.

- What is the domain of the function $(x) = (x + 2)^{\frac{1}{2}}$?

$$f(x) = (x + 2)^{\frac{1}{2}} = \sqrt{x + 2}$$

  Because we can't take the square root of a negative number, we know that $x + 2$ can't be negative:

$$x + 2 \geq 0$$
$$x \geq -2$$

- What number is not in the domain of the function $f(x) = \frac{1}{x^2}$?

  We can take the square of any real number x, but we can't divide by zero. Therefore:

$$x^2 \neq 0$$
$$x \neq 0$$

Zero is the only number not in the domain of our function.

Instead of using function notation to represent the relationships between sets of numbers, the SAT will often use **strange symbols**: smiley faces, stars, bubbles, etc. Treat these the same way you would treat function questions—look for the rule, and figure out how you should apply it.

- For all negative values *x*, let $✧x = 4x + 7$. For zero and all positive values *x*, let $✧x = 3x^2$. What is the value of ✧✧(-1)?

$$✧(-1) = 4(-1) + 7 = 3$$
$$✧(3) = 3(3^2) = 27$$
$$✧✧(-1) = 27$$

- For all non-zero numbers *a* and *b*, let △ be defined as

$$a \bigtriangleup b = \frac{2a}{b}$$

If $3 \bigtriangleup 4 = x \bigtriangleup 2$, what is the value of *x*?

$$3 \bigtriangleup 4 = \frac{2 \times 3}{4} = \frac{6}{4}$$
$$x \bigtriangleup 2 = \frac{2x}{2} = x$$
$$\frac{6}{4} = x$$
$$x = \frac{3}{2}$$

- Let ☼x represent the number of factors of any real number *x*. For example, ☼$6 = 4$ because 6 has four factors: 1, 2, 3 and 6. What is ☼9?

9 has three factors: 1, 3, and 9. Therefore, ☼$9 = 3$.

The **graph** of a function is the collection of all ordered pairs $(x, y)$where $y = f(x)$. Thus, the graph of $f(x) = x + 1$ is the line $y = x + 1$. You will never be asked to draw a graph on the SAT, but you may need to derive an equation or function based on a graph.

Linear functions can be represented in the form $y = mx + b$, where m is the slope and b is the y-intercept. Remember the formula for calculating slope:

$$m = \frac{\text{rise}}{\text{run}} = \frac{y_2 - y_1}{x_2 - x_1}$$

Facts about slope:

- **Horizontal lines** have slopes of zero.
- **Vertical lines** have undefined slopes.
- Non-vertical **parallel lines** have equal slopes.
- Non-vertical **perpendicular lines** have slopes whose product is -1.

- A line with a slope of 5 passes through the points (k, 7) and (-1, -3). What is the value of k?

$$5 = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-3 - 7}{-1 - k}$$
$$5(-1 - k) = -10$$
$$-1 - k = -2$$
$$k = 1$$

- In the xy-coordinate plane, line h has a slope of 2 and is perpendicular to line g. If line g passes through the origin and intersects line h at the point (a, 3 + a), what is the value of a?

  Let m represent the slope of line g. Line g and line h are perpendicular, so:

$$2m = -1$$
$$m = -\frac{1}{2}$$

  We know that line g passes through the origin, so its y-intercept is 0. Its equation must be:

$$y = -\frac{1}{2}x$$

  Plug the point (a, 3 + a) into this equation to solve for a:

$$3 + a = -\frac{1}{2}a$$
$$\frac{3}{2}a = -3$$
$$a = -2$$

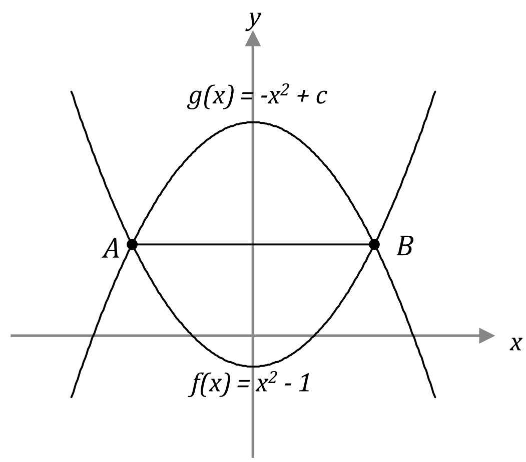
Quadratic functions can be represented in the form $y = ax^2 + bx + c$, where c is the y-intercept. The sign of a determines whether the parabola opens upwards (a is positive) or downwards (a is negative). You can often find the x-intercepts of a parabola by factoring; the SAT will never require you to use the quadratic formula.

The **vertex** of a parabola is its lowest or highest point. If we call the coordinates of the vertex (h, k), then:

$$h = \frac{-b}{2a}$$

When asked to solve for other points on a parabola, remember that a parabola is **symmetrical** about the vertical line through its vertex.

- In the figure below, $f(x) = x^2 - 1$ and $g(x) = -x^2 + c$, where c is a constant. The two functions intersect at points A and B. If $\overline{AB} = 4$, what is the value of c?



Because a parabola is symmetrical and $\overline{AB} = 4$, each point of intersection must be 2 units away from the y-axis. Thus, we can call their coordinates (-2, y) and (2, y). Solve for y by plugging one of these points into the first equation:

$$f(x) = x^2 - 1$$
$$y = 2^2 - 1$$
$$y = 3$$

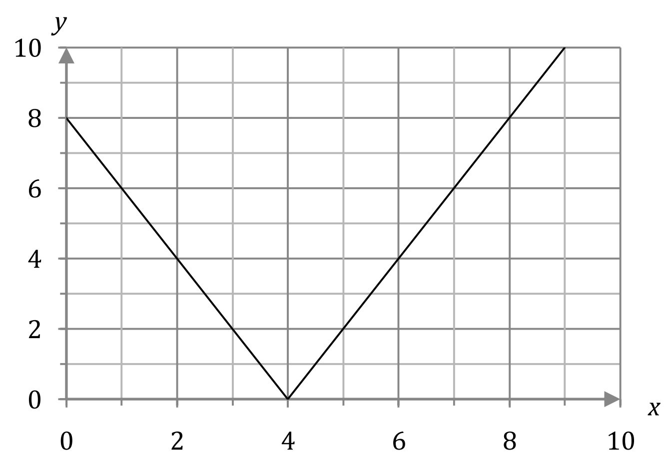
The two equations intersect at (-2, 3) and (2, 3). Plug one of these points into the second equation to solve for c:

$$g(x) = -x^2 + c$$
$$3 = -2^2 + c$$
$$c = 7$$

You may be asked to analyze the behaviour of graphs without knowing or using their equations.

- The graph of $y = f(x)$ is shown below. If $f(2) = d$, what is $f(d)$?

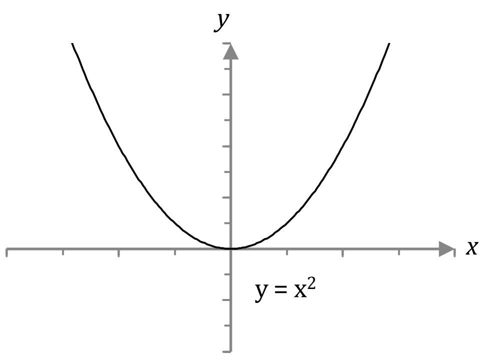
$$f(2) = 4, \text{so } d = 4$$
$$f(d) = f(4) = 0$$

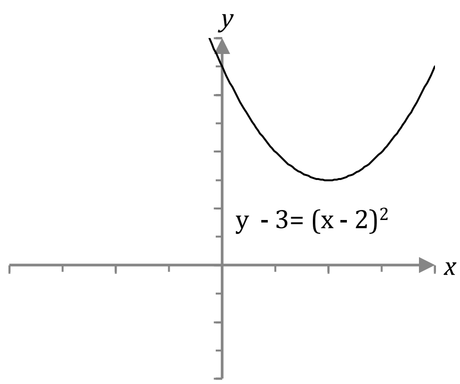


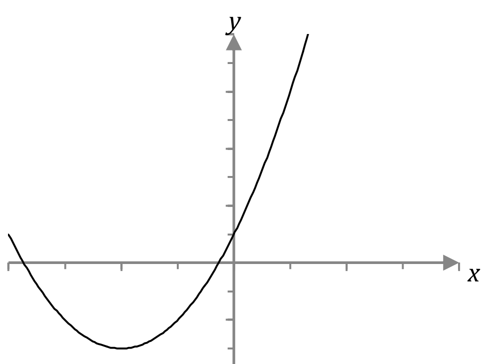
A **translation** of a graph is a rigid shift vertically or horizontally that does not change the size or shape of the graph. For the graph of $y = f(x)$, a vertical translation of a units and a horizontal translation of b units results in the equation

$$y - a = f(x - b)$$

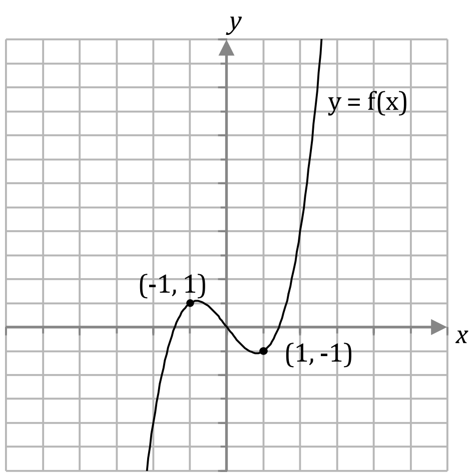
Let's begin with the graph of $y = x^2$. If we shift vertically 3 units and horizontally 2 units, we get $y - 3 = (x - 2)^2$. If we shift vertically -3 units and horizontally -2 units, we get $y + 3 = (x + 2)^2$.







- Below are the graphs of the functions $f(x) = x^3 - 2x$ and $g(x) = f(x - c) + d$, where c and d are both constants. What is the value of $d + c$?



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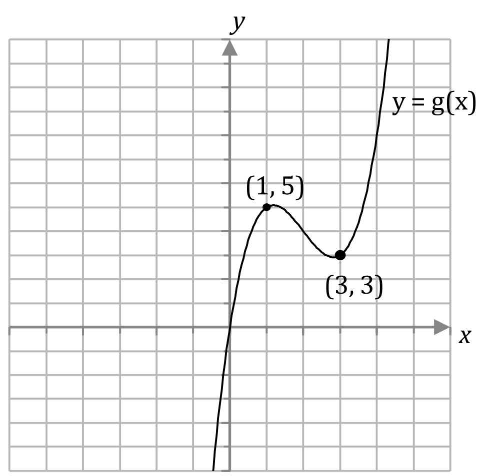


In the equation $y - a = f(x - b)$, a represents the vertical shift and b represents the horizontal shift. Re-write the equation of *g(x)* to look similar:

$$g(x) - d = f(x - c)$$

Here, *d* represents the vertical shift and *c* represents the horizontal shift. By comparing the two points given on the graph, we can see that the graph has been translated 2 units horizontally and 4 units vertically. Thus:

$$c = 2$$
$$d = 4$$
$$d + c = 6$$

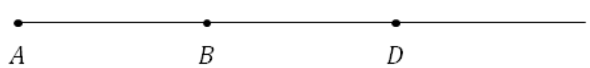
III. Geometry

- ***Points, lines, and angles***
- ***Triangles and special triangles***
- ***Polygons and quadrilaterals***
- ***Circles***
- ***Solid geometry***
- ***Coordinate geometry***

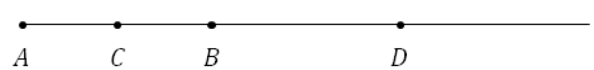
Any two distinct points can be connected by one and only one line. A **midpoint** is the point that divides a line segment into two equal parts. You may be asked to find the length of a line segment or the ratio of one line segment to another. You may also need to find the order of points along a common line.

- Points *A, B, C, D,* and *E* lie on a line, not necessarily in that order. *B* is the midpoint of $\overline{AD}$, *C* is the midpoint of $\overline{AB}$, and *D* is the midpoint of $\overline{BE}$. If $AC = 4$, what is the value of CE?

  Begin by drawing a diagram to put the points in the correct order. *B* is the midpoint of $\overline{AD}$:

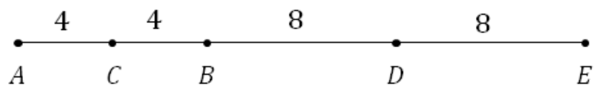


  C is the midpoint of $\overline{AB}$:



  D is the midpoint of $\overline{BE}$:

We know that $AC = 4$, so we can write in the length of each line segment:



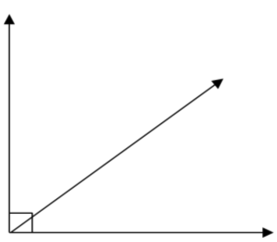
Adding together the component line segments, we find that $CE = 20$.

An angle is formed when two lines or line segments intersect. Know the following definitions:

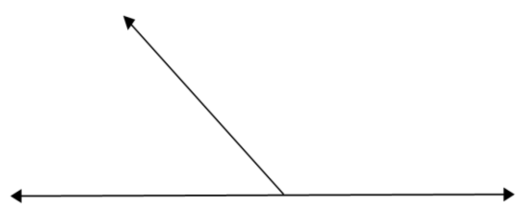
- An angle measuring less than 90° is an **acute** angle.
- An angle measuring 90° is a **right** angle.
- An angle measuring between 90° and 180° is an **obtuse** angle.
- An angle measuring 180° is a **straight** angle.

Know also the following angle sums:

- The sum of any number of angles that form a straight line is 180°.
- The sum of any number of angles around a point is 360°.
- Two angles that add to 90° are called **complementary** angles.
- Two angles that add to 180° are called **supplementary** angles.

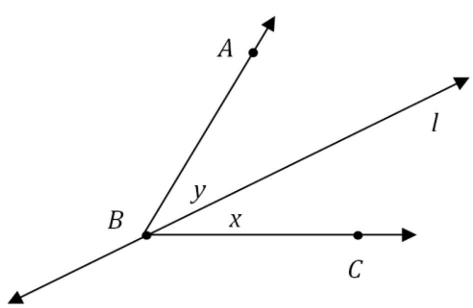


Complementary Angles

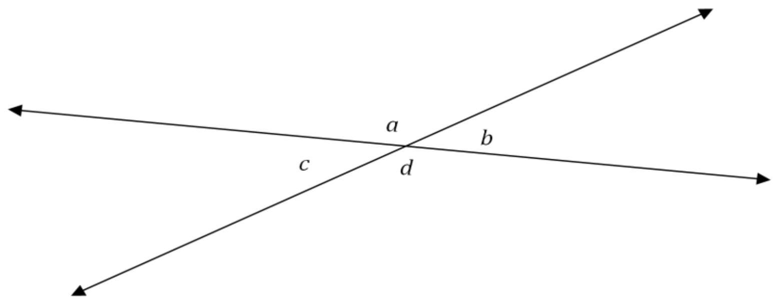


Supplementary Angles

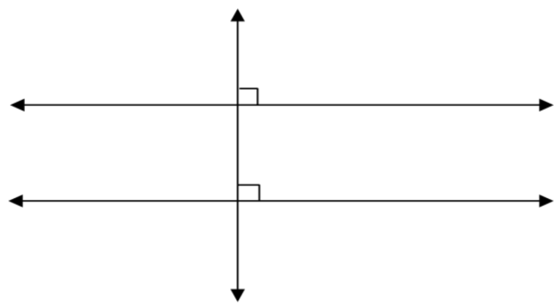
A line that **bisects** an angle divides it into two equal parts. In the following diagram, if $x = y$, then line *l* bisects $\angle ABC$:



Vertical angles are the opposite angles formed by two intersecting lines. These are **congruent**: $a = d$ and $b = c$.

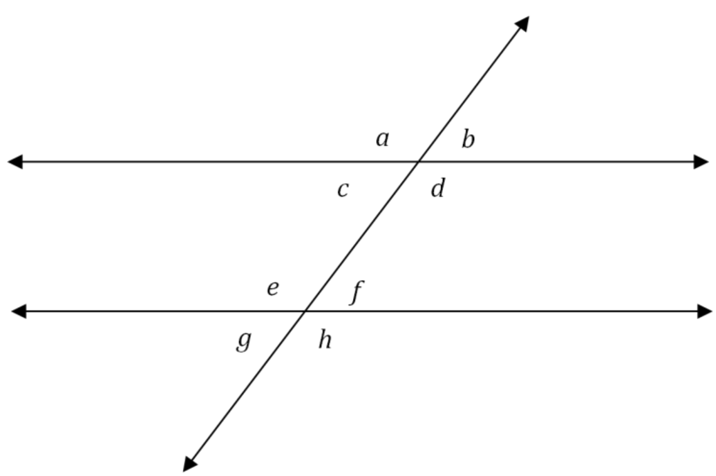


If two lines are **parallel**, they will never intersect. We can determine that two lines are parallel if there is a third line that is perpendicular to both of them:

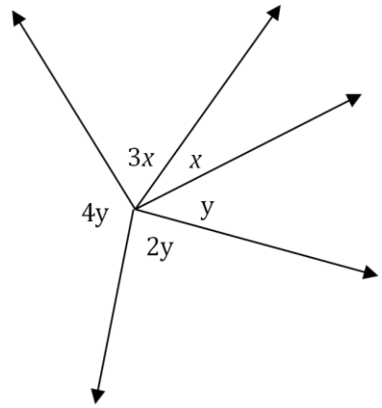


If a third line (**transversal**) intersects a pair of parallel lines, it forms eight angles. Know the following relationships:

- The pairs of **corresponding** angles are congruent: $a = e,\ b = f,\ c = g,$ and $d = h$.
- The pairs of **alternate interior** angles are congruent: $c = f$ and $d = e$.
- The pairs of **alternate exterior** angles are congruent: $a = h$ and $b = g$.
- The pairs of **consecutive interior** angles are supplementary: $c + e = 180°$ and $d + f = 180°$.



- In the diagram below, five lines intersect in a point to form five angles. If $y = 2x$, what is the value of x?



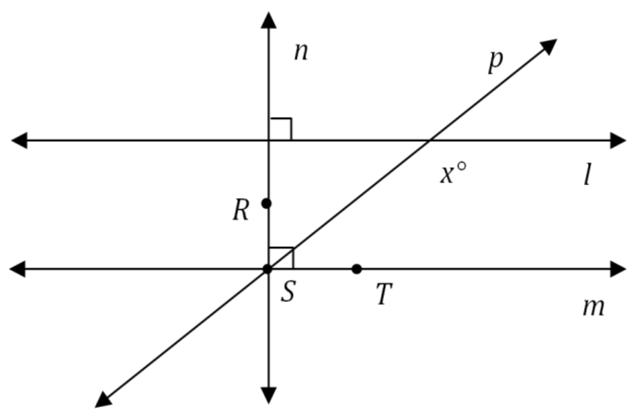
The sum of angles around a point is 360°:

$$x + 3x + y + 2y + 4y = 360$$
$$4x + 7y = 360$$

$y = 2x$, so we can substitute and solve for x:

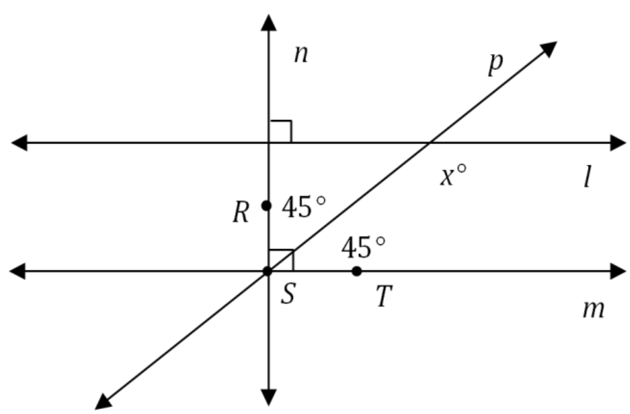
$$4x + 14x = 360$$
$$18x = 360$$
$$x = 20$$

- In the diagram below, line n is perpendicular to both line l and line m, and line p bisects $\angle RST$. What is the value of x?



Because line n is perpendicular to both l and m, we can conclude that l and m are parallel lines.

Line p bisects $\angle RST$, a right angle, and divides it into two angles of 45° each. We can label these on the diagram.

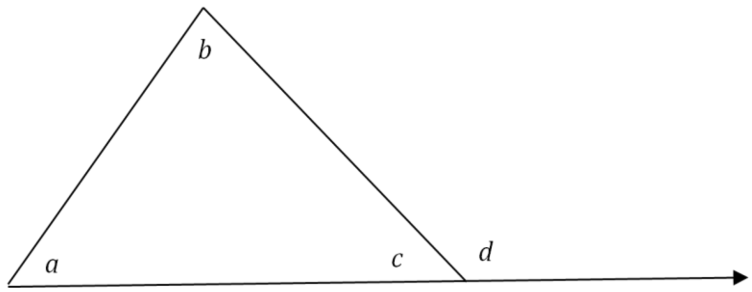


Because line p intersects two parallel lines, we know that pairs of consecutive interior angles are supplementary. Therefore:

$$x + 45 = 180$$
$$x = 135$$

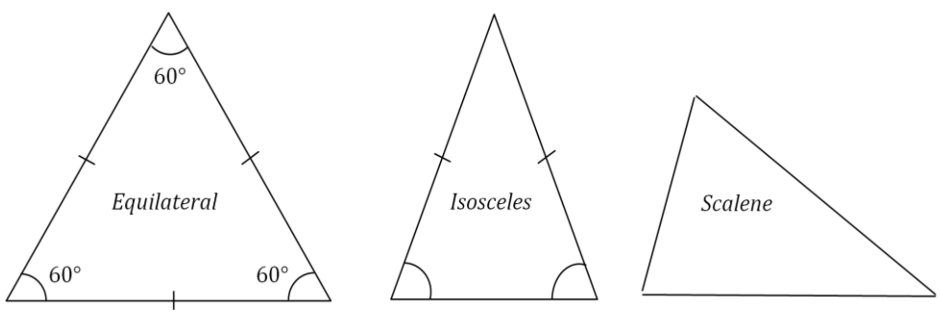
Triangle geometry is the most commonly tested geometry topic on the SAT.

The sum of the interior angles of a triangle is 180°. An **exterior angle** is an angle that is formed by extending one side of the triangle. An exterior angle of a triangle is equal to the sum of the two opposite interior angles. In the following diagram, $a + b + c = 180$ and $d = a + b$.



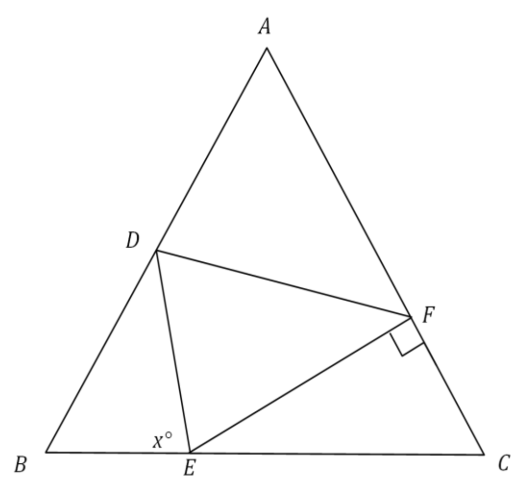
Triangles can be classified by interior angles and side lengths:

- An **acute** triangle has three acute angles.
- An **obtuse** triangle has one obtuse and two acute angles.
- A **right** triangle has one angle measuring 90°.
- An **equilateral** triangle has three congruent sides and three congruent angles. All three of these angles measure 60°.
- An **isosceles** triangle has two congruent sides and two corresponding congruent angles.
- A **scalene** triangle has three sides of different lengths and three angles of different measures.



- In the figure to the right, ΔABC is an equilateral triangle, ΔEFC is a right triangle, and $DF = EF$. If the measure of $\angle DFE$ is 20°, what is the value of x?

  Fill in as many of the angle measures as possible with the given information. ΔABC is an equilateral triangle, so each of its interior angles measures 60°.

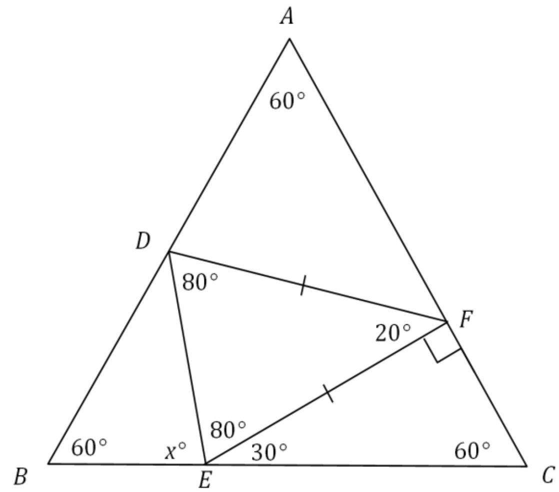


$DF = EF$, so ΔDFE is isosceles and $\angle FDE \cong \angle FED$. Because $\angle DFE$ measures 20° and the interior angles of a triangle must add to 180°, we can conclude that $\angle FDE$ and $\angle FED$ each measure 80°.

Finally, $\angle EFC$ measures 90° and $\angle ECF$ measures 60°, so $\angle CEF$ must measure 30°.

We now have all the information necessary to solve for x, which lies on a straight line with two other angles:

$$x + 80 + 30 = 180$$
$$x = 70$$

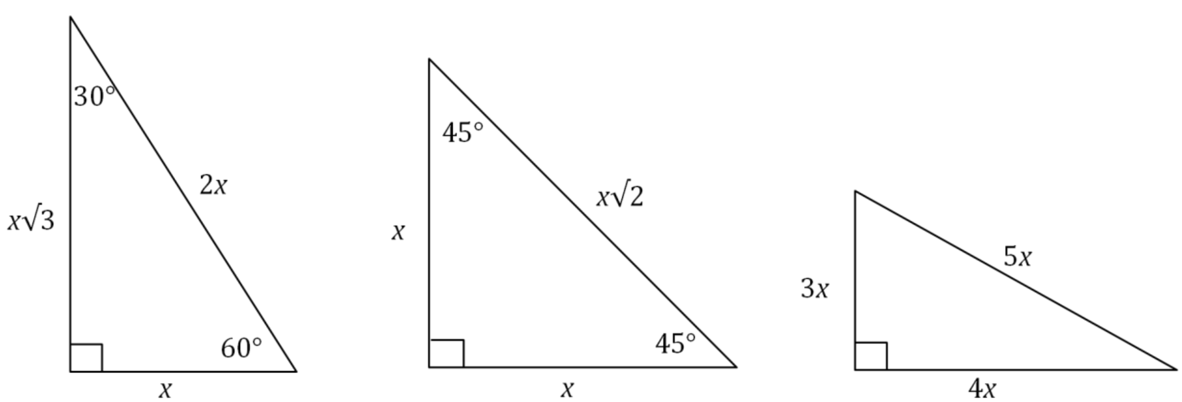


The **hypotenuse** of a right triangle is the side opposite the right angle. If the two legs of a right triangle have lengths a and b and the hypotenuse has a length c, then the **Pythagorean Theorem** states that

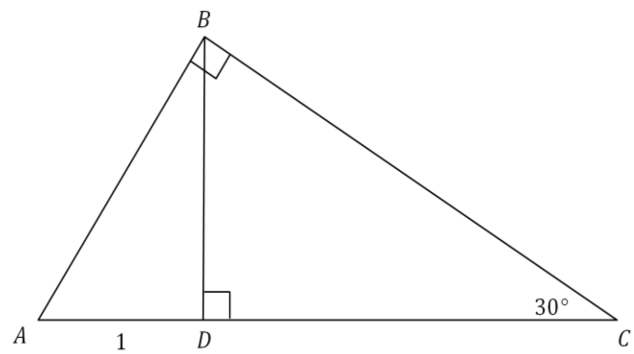
$$a^2 + b^2 = c^2$$

Special right triangles have a set ratio among the lengths of their sides, derived from the Pythagorean Theorem.

- A triangle with angles of **30°-60°-90°** has a short leg measuring x, a long leg measuring $x\sqrt{3}$, and a hypotenuse measuring $2x$.
- A triangle with angles of **45°-45°-90°** has two legs that each measure x and a hypotenuse measuring $x\sqrt{2}$.
- A **3-4-5** right triangle is a right triangle whose sides are in the ratio 3:4:5.



- In the diagram to the right, ΔABC is a right triangle, $\angle BDC$ is a right angle, $\angle DCB$ measures 30°, and $AD = 1$. What is the perimeter of ΔABC?

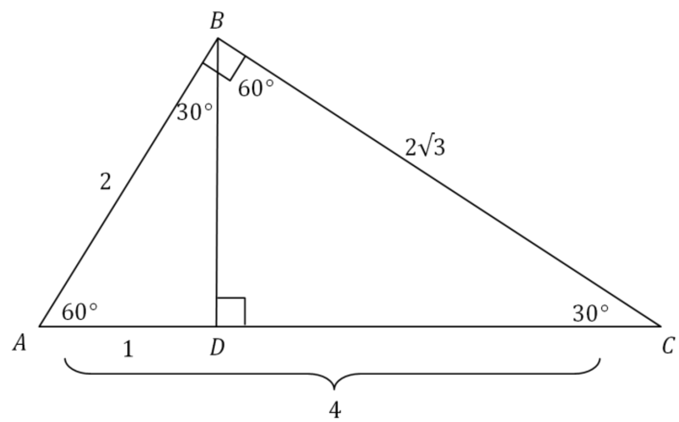
  

  ΔABC, ΔADB, and ΔBDC are all 30°-60°-90° triangles. Because the length of AD is given, we can find the lengths of AB:

  $$AD = 1$$
  $$AB = 2$$

  From the length of AB, we can find the lengths of BC and AC:

  $$AB = 2$$
  $$BC = 2\sqrt{3}$$
  $$AC = 4$$

  The perimeter of ΔABC is $2 + 2\sqrt{3} + 4 = 6 + 2\sqrt{3}$.

The **triangle inequality** states that the sum of the lengths of two sides of a triangle is always greater than the length of the third side.

- If two sides of a triangle measure 4 and 6 units, what could be the length of the third side?

  Using the triangle inequality, we know that the length of third side must be less than 10 units:

  $$4 + 6 > x, \text{ so } 10 > x$$

  We also know that the lengths of the third side plus either one of the other sides must be greater than the length of the remaining side:

  $$x + 4 > 6, \text{ so } x > 2$$
  $$x + 6 > 4, \text{ so } x > -2$$

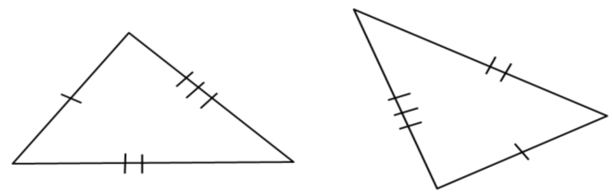
  The length of the third side must be greater than 2 but less than 10 units.

If b represents a triangle's base and h represents its height, the **area** of a triangle is

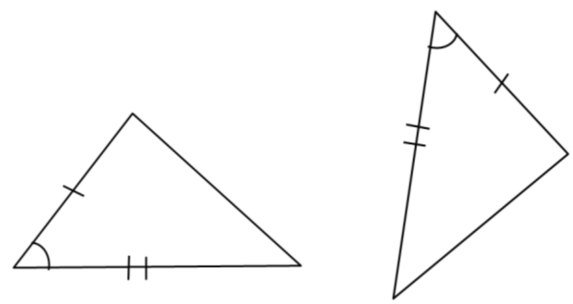
$$A = \frac{1}{2}bh$$

Congruent triangles are the same shape and size: they have corresponding congruent angles and congruent sides. We can prove that two triangles are congruent by any of the following means:

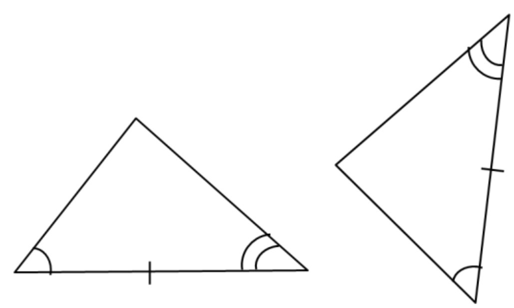
- **Side-side-side***:* the corresponding sides are congruent



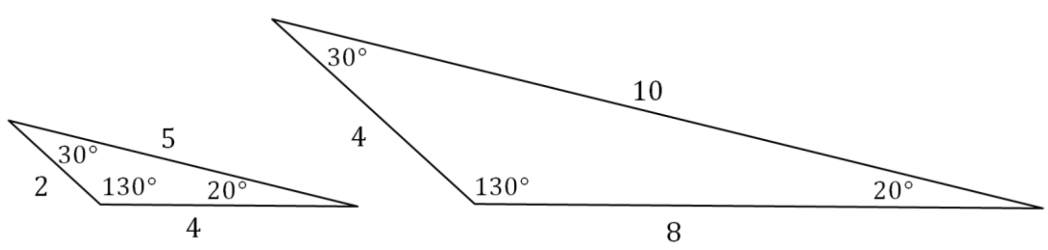
- **Side-angle-side**: two corresponding sides and the angles they form are congruent



- **Angle-side-angle**: two corresponding angles and the sides between them are congruent



Similar triangles have the same shape but different sizes. Their corresponding angles are congruent, and their corresponding sides are proportional. We can prove two triangles are similar if two corresponding angle pairs are congruent, or if one corresponding angle pair is congruent and the adjacent sides are proportional.



Similar Triangles

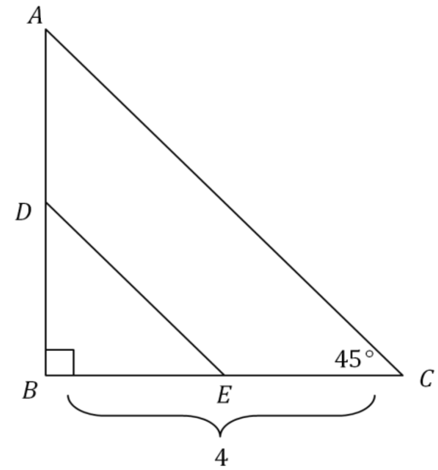
- In the diagram to the right, ΔABC is a right triangle, $\angle C$ measures 45°, and $BC = 4$. If D is the midpoint of $\overline{\text{AB}}$ and E is the midpoint of $\overline{\text{BC}}$, what is the length of DE?

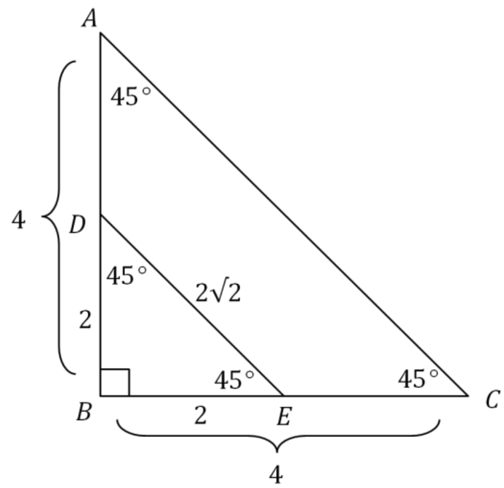
  All interior angles of a triangle add to 180°, so $\angle A$ also measures 45°. ΔABC a 45°-45°-90° triangle and an isosceles triangle, so $AB = 4$ as well.

  D is the midpoint of $\overline{\text{AB}}$ and E is the midpoint of $\overline{\text{BC}}$, so BE and BD each measure 2 units. Because BE and BD are each half of AB and BC, and both pairs form the same right angle, we can conclude that ΔABC and ΔDBE are similar triangles. Thus, ΔDBE is also a 45°-45°-90° triangle.

  If each of the legs of ΔDBE measure 2 units, then the hypotenuse must measure $2\sqrt{2}$ units.

$$DE = 2\sqrt{2}$$



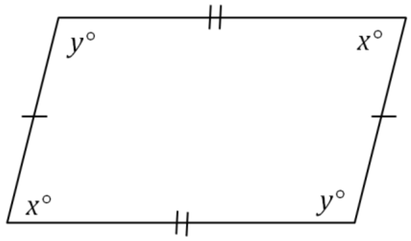


A **polygon** is an enclosed shape with straight sides. A **regular** polygon has congruent sides and congruent interior angles. For a polygon with n sides, the sum of the interior angles is

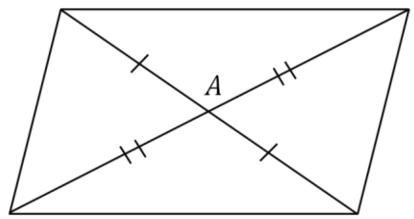
$$180°(n - 2)$$

A **quadrilateral** is a four-sided polygon. The sum of the interior angles of a quadrilateral is 360°.

A **parallelogram** is a quadrilateral with two pairs of parallel sides. Rhombuses, rectangles, and squares fall into this category. The opposite sides of a parallelogram are congruent in length and the opposite angles are congruent in measure. Adjacent angles are supplementary. Each diagonal divides the parallelogram into two congruent triangles. The two diagonals are *not* necessarily the same length, but they bisect each other.



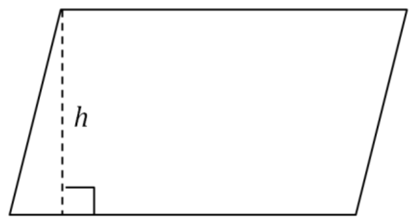
$x° + y° = 180°$



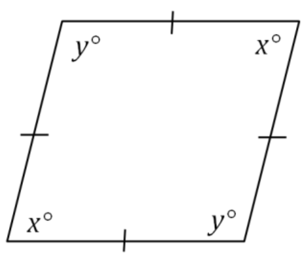
A is the midpoint of both diagonals

If *b* represents the base of the parallelogram and *h* represents its height (perpendicular to the base), then the formula for the area of any parallelogram is

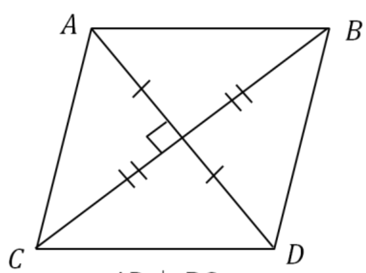
$$A = bh$$



A **rhombus** is a quadrilateral whose four sides are all the same length. All rhombuses are also parallelograms and have the same properties. Additionally, the diagonals of a rhombus are always perpendicular and bisect their interior angles.

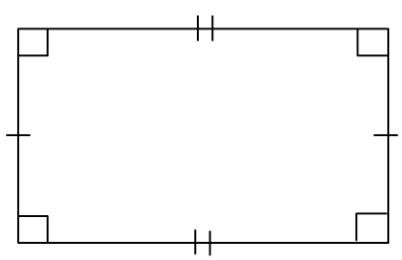


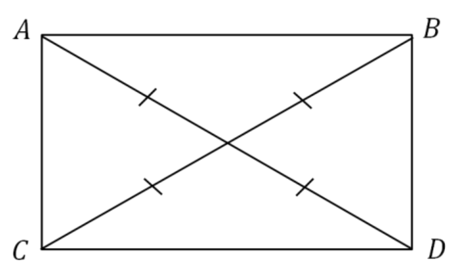
$x° + y° = 180°$



$AD \perp BC$

A **rectangle** is a quadrilateral with four right angles. All rectangles are also parallelograms and have the same properties, but the diagonals of a rectangle are the same length.

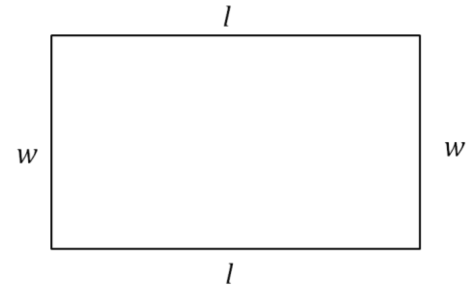




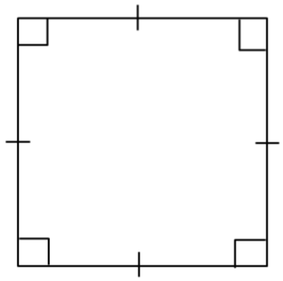
$AD = BC$

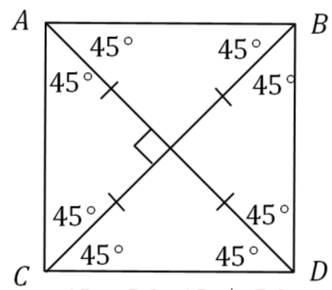
The area of a rectangle is simply its base multiplied by its height. If its length is l and its width is w, then the perimeter of a rectangle is

$$P = 2(l + w)$$



A **square** is a **regular** quadrilateral—that is, all four sides and all four angles are the same measure. Squares are both rectangles and rhombuses. The diagonals of a square are of equal length and perpendicular, and they bisect its interior angles. The diagonals divide the square into congruent 45°-45°-90° triangles.

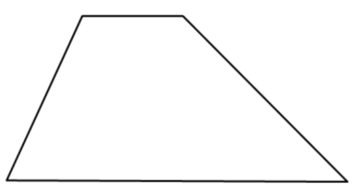




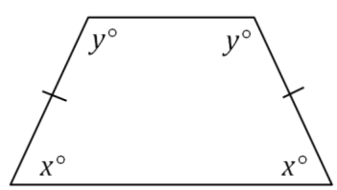
$AD = BC,\ AD \perp BC$

If the length of a side is s, then the area of a square is s^2 and the perimeter is $4s$.

A **trapezoid** is a quadrilateral with only one pair of parallel sides. An **isosceles** trapezoid has two pairs of congruent angles, and its two non-parallel sides are congruent.



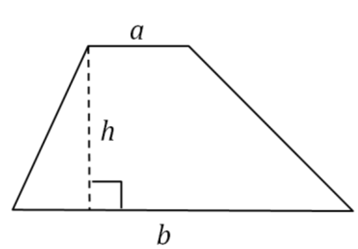
Trapezoid



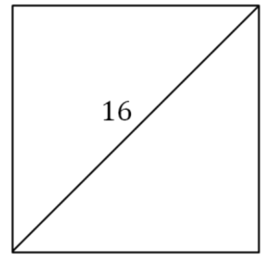
Isosceles Trapezoid

If a and b are the lengths of the parallel sides and h is the height perpendicular to these sides, then the area of a trapezoid is

$$A = \frac{1}{2}h(a + b)$$

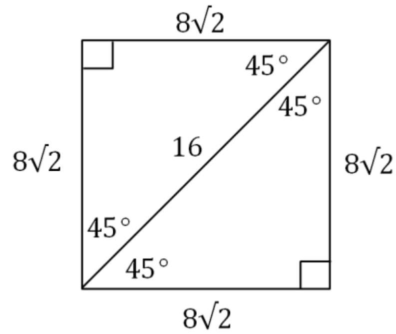


- What is the area of the square shown to the right?

  We know that a diagonal of a square divides it into 45°-45°-90° triangles, with sides of length x and a hypotenuse of length $x\sqrt{2}$. Here, the hypotenuse is 16 units long, so we can find the length of each side:

  $$x\sqrt{2} = 16$$
  $$x = \frac{16}{\sqrt{2}} = \frac{16\sqrt{2}}{2} = 8\sqrt{2}$$

  If each side is $8\sqrt{2}$ units long, then the area of the square is

  $$(8\sqrt{2})^2 = 128$$

- In the quadrilateral $ABCD$ to the right, $\overline{AB}$ is parallel to $\overline{CD}$, $\overline{BD}$ is perpendicular to $\overline{CD}$, and $\angle C$ measures 60°. If $BD = 4\sqrt{3}$ and $AB = 2$, what is the area of quadrilateral $ABCD$?

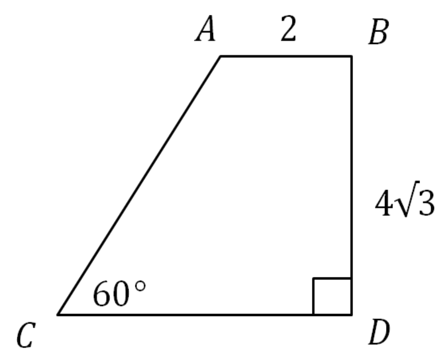
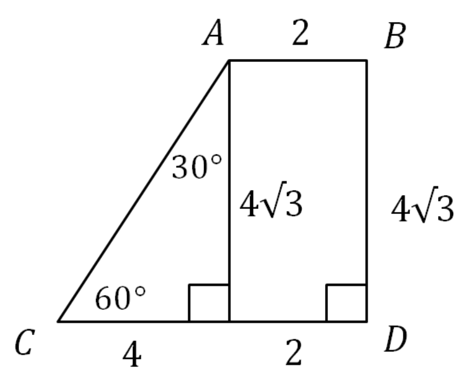
  

  Figure not necessarily drawn to scale

  If we drop another line from point A perpendicular to $\overline{CD}$, we can break up the trapezoid into a rectangle and a triangle. The rectangle has side lengths 2 and $4\sqrt{3}$, and the triangle is a 30°-60°-90° triangle with a long side of length $4\sqrt{3}$. Because we know that a 30°-60°-90° triangle has a short side of length x and a long side of length $x\sqrt{3}$, we can solve for the short side:

  $$x\sqrt{3} = 4\sqrt{3}$$
  $$x = 4$$

  We now know that the trapezoid $ABCD$ has a short base of length 2, a long base of length 6, and a height of $4\sqrt{3}$. The area of the trapezoid is

  $$A = \frac{1}{2}h(a + b)$$
  $$A = \frac{1}{2}\left(4\sqrt{3}\right)(2 + 6) = 16\sqrt{3}$$

A **circle** is a set of all points in a plane that are the same distance from a given point. This distance, from the centre of the circle to any point on the circle, is called the **radius.** All radii of one circle are the same length. A line connecting two points on the circle and passing through the centre is called a **diameter.** The diameter of a circle is equal to twice the length of the circle's radius.

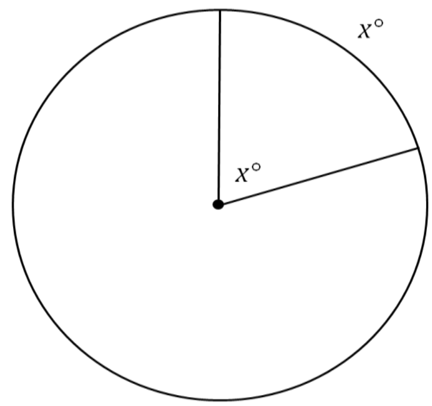
If d is the length of the diameter and r is the length of the radius, the **circumference** of a circle is

$$C = \pi d = 2\pi r$$

If r is the length of the radius, the **area** of a circle is

$$A = \pi r^2$$

An **arc** is a segment of the circumference of a circle. Arcs are measured either in degrees or in units of length. Drawing two radii from the endpoints of the arc to the centre of the circle creates the arc's **central angle**, whose degree measure is equal to the degree measure of the arc. The ratio between the length of the arc and the circumference of the circle is equal to the ratio between the degrees in the central angle and the degrees in the entire circle. If the arc's central angle measures $x°$ and the arc's length is L, then:

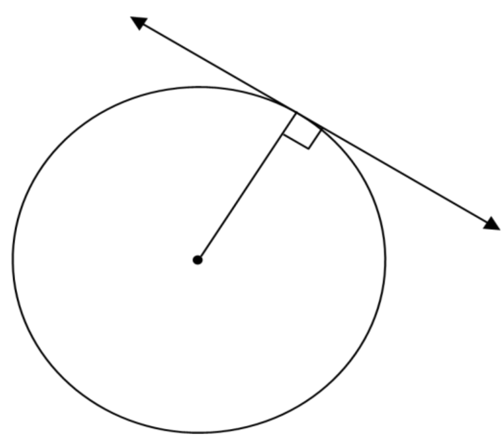


$$\frac{L}{2\pi r} = \frac{x°}{360°}$$

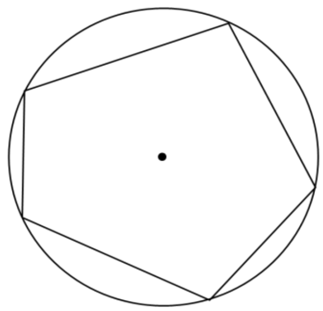
A **sector** is a region of a circle bounded by two radii and an arc. The ratio between the area of a sector and the area of the entire circle is also equal to the ratio between the degrees in the central angle and the degrees in the entire circle. If the sector's central angle measures $x°$ and the sector's area is A, then:

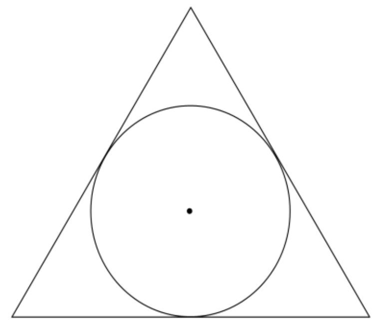
$$\frac{A}{\pi r^2} = \frac{x°}{360°}$$

A line that intersects a circle at exactly one point is called a **tangent**. The radius drawn from the point of intersection to the centre of the circle is always perpendicular to the tangent.

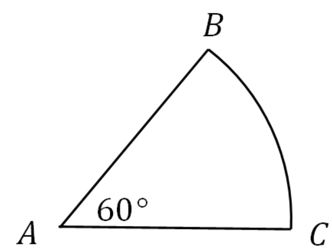


If every vertex of a polygon lies on a circle, then the polygon is said to be **inscribed** in the circle. If each side of a polygon is tangent to a circle, then the circle is inscribed in the polygon.

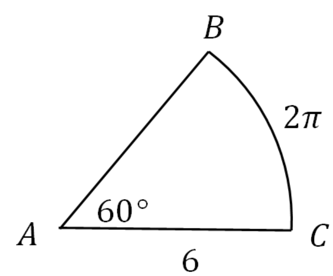




- In the figure to the right, BC is an arc of a circle whose centre is A. If the length of arc BC is 2π units, what is the area of sector ABC?



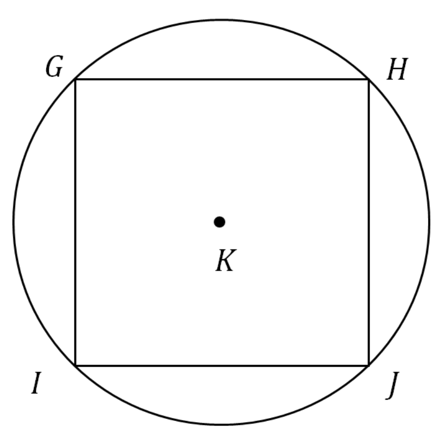
The central angle of arc BC is 60°, which is $\frac{60}{360} = \frac{1}{6}$ of the total degrees of the circle. The length of the arc is 2π units, which must be $\frac{1}{6}$ the total circumference of the circle. Thus, the circumference must be 12π units. We can use the circumference to solve for the radius:



$$C = 2\pi r$$
$$12\pi = 2\pi r$$
$$r = 6$$

If the radius is 6 units, the area of the entire circle is 36π units squared. The area of sector ABC is $\frac{1}{6}$ the total area of the circle, so the area of ABC must be 6π units squared.

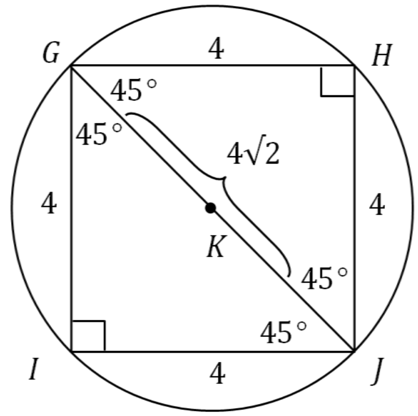
- In the figure to the right, square $GHIJ$ is inscribed in a circle whose centre is K. If the area of square $GHIJ$ is 16, what is the area of circle K?



The area of the square is 16, so the length of each side must be $\sqrt{16} = 4$.

Drawing a diagonal of the square divides it into two 45°-45°-90° triangles. If each side of these triangles has a length of 4, then the length of the hypotenuse must be $4\sqrt{2}$.

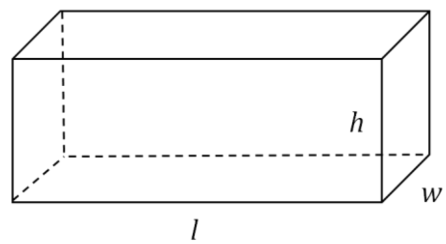
This hypotenuse is also a diameter of circle K. If its length is $4\sqrt{2}$, then the radius of circle K has a length of $2\sqrt{2}$. The area of circle K is $\pi(2\sqrt{2})^2 = 8\pi$.



Solid geometry is concerned with figures in three dimensions: rectangular solids, prisms, cylinders, pyramids, cones and spheres. Though all of these solids may appear on the SAT, you will primarily be working with rectangular solids, prisms, and right cylinders.

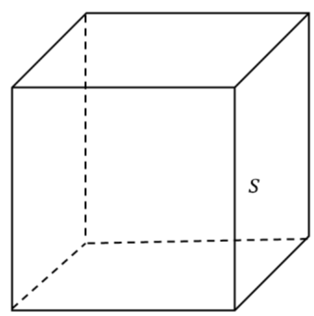
A **rectangular solid** has six rectangular faces that intersect at right angles. The surface area of a rectangular solid is the sum of the areas of all six of its rectangular faces. The volume of a rectangular solid is its length multiplied by its width multiplied by its height.

$$V = lwh$$
$$SA = 2lw + 2wh + 2lh$$



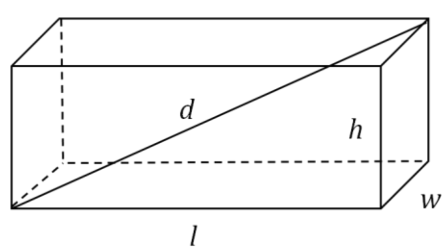
A **cube** is a rectangular solid formed by six congruent squares—the length, width, and height are equal. If s is the length of the cube's side, then

$$V = s^3$$
$$SA = 6s^2$$

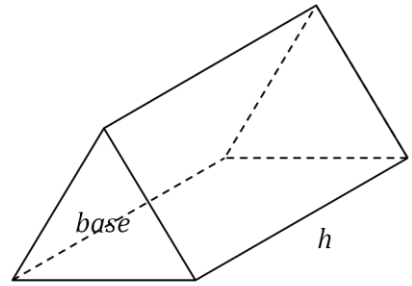


The **diagonal** of a rectangular solid joins opposite vertices—it is the longest line that can be drawn through the solid. For a rectangular solid with length l, width w, height h, and diagonal d:

$$d^2 = l^2 + w^2 + h^2$$

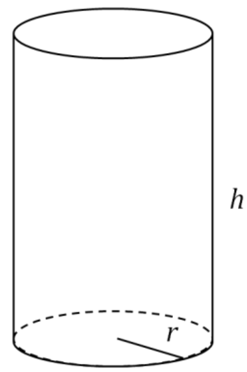


A **prism** is any solid with two congruent bases joined by perpendicular rectangles. A rectangular solid is a type of prism. A triangular prism has triangles as bases, an octagonal prism has octagons as bases, and so on. The volume of a prism equals the area of its base multiplied by its height.



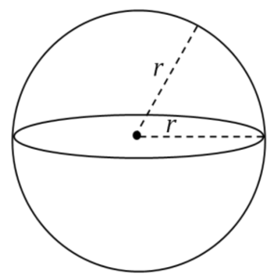
A **right cylinder** is similar to a prism: it has two circular bases connected by a perpendicular curved surface. The volume of a cylinder is equal to the area of its base multiplied by its height. The surface area of a cylinder is the sum of the areas of its bases and the area of the curved rectangle that connects them; the two sides of this rectangle are the cylinder's height and the circumference of the base. If r is the radius of a cylinder's base and h is its height, then

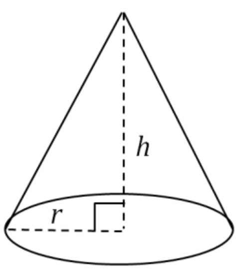
$$V = \pi r^2 h$$
$$SA = 2\pi r^2 + 2\pi r h$$



Other solids incude spheres, cones, and pyramids. You will not be requred to memorize the formulas for volume and surface area of these solids.

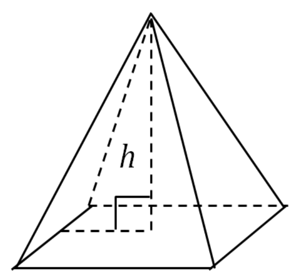
A **sphere** is the collection of all points in space that are the same distance away from a centre point. All radii of a sphere are equal.



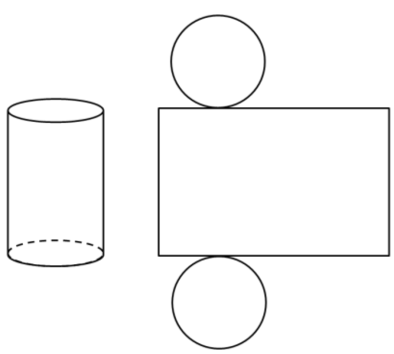


A **cone** has a circular base and a curved surface that tapers to a point, called the vertex. In a **right circular cone**, the line connecting the vertex to the centre of the base forms a right angle with the base.

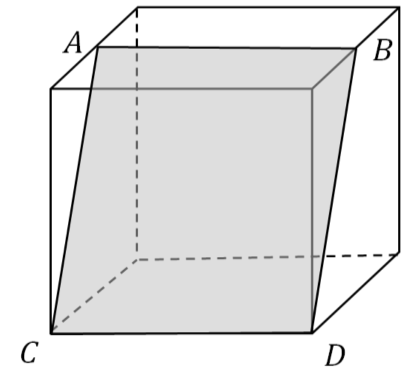
A **pyramid** has a polygon for a base and triangular faces that join in a point, called the vertex. A **regular pyramid** has a regular polygon for its base and congruent isosceles triangles for its sides.



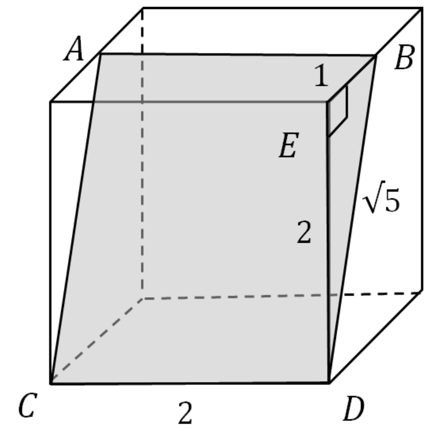
A **net** is a figure in a plane formed by "unfolding" a solid along its edges. Nets can help you calculate surface area. The following is the net of a cylinder:



- The figure to the right shows a cubic box with a side length of 2. A rectangular piece of metal has been inserted into the box at a slant, so the bottom edge CD intersects the inside front edge of the cube. If points A and B are midpoints of the top edges of the cube, what is the area of the rectangle $ABCD$?



  We know the length of AB and CD are each 2, but we need to find the lengths of AC and BD. We can see that BD forms a right triangle with two of the edges of the cube. We'll call the third point on this triangle point E.



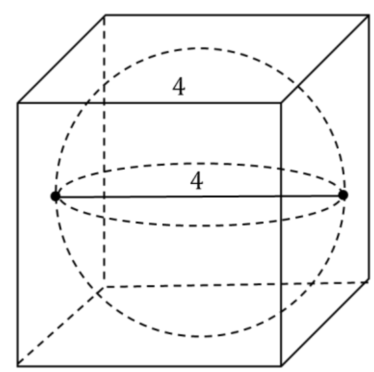
  The side length of the cube is 2, so $DE = 2$. We know that B is the midpoint of one of the upper sides, so $BE = 1$. We can solve for BD using the Pythagorean Theorem:

$$(BD)^2 = 2^2 + 1^2$$
$$BD = \sqrt{5}$$

  The area of rectangle $ABCD$ is $2 \times \sqrt{5} = 2\sqrt{5}$.

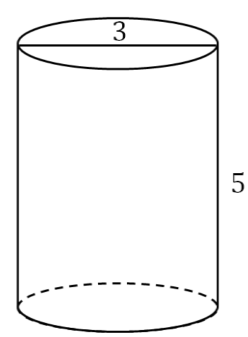
- A spherical ball is completely inscribed within a cube whose volume is 64 cubed inches. What is the radius of the ball?

  Let's draw a figure. If the sphere is inscribed within the cube, the diameter of the sphere is equal to the width of the cube. We know the cube's volume is 64 cubed inches, so the length of each side of the cube is $\sqrt[3]{64} = 4$ inches. The diameter of the sphere is 4 inches, so its radius must be 2 inches.



- A hollow cylinder 5 centimeters long is formed so that the circular opening on each side has a diameter of 3 centimeters. The outside of the cylinder is to be painted red. What is the total surface area of paint needed?

  The outside of the cylinder is simply a rectangle whose dimensions are equal to the height of the cylinder and the circumference of the circular base. The height is 5 centimeters long, and the



circumference of the base is $\pi d = 3\pi$ centimeters. The area of the surface to be painted is $5 \times 3\pi = 15\pi$ centimeters squared.

Coordinate geometry deals with figures in the coordinate plane. You may need to calculate the slope or length of lines, or the area or perimeter of polygons given their coordinate points.

Keep in mind (from the Algebra and Functions section):

- **Horizontal lines** have slopes of zero.
- **Vertical lines** have undefined slopes.
- Non-vertical **parallel lines** have equal slopes.
- Non-vertical **perpendicular lines** have slopes whose product is -1.

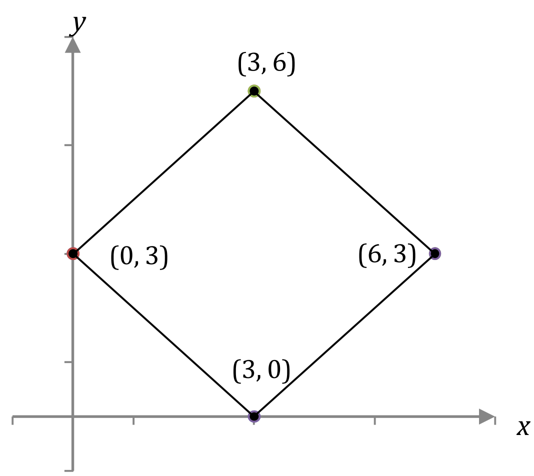
We can apply the Pythagorean Theorem to find the **distance** between two points in the coordinate plane. If two points have coordinates (x_1, y_1) and (x_2, y_2), then the distance *d* between them is:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

If two points have coordinates (x_1, y_1) and (x_2, y_2), then the coordinates of **midpoint** of the segment connecting them is simply the average of their coordinates:

$$(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2})$$

- A square lies in the *xy*-coordinate plane with vertices (0,3), (3,0), (3,6) and (6,3). What is the area of the square?



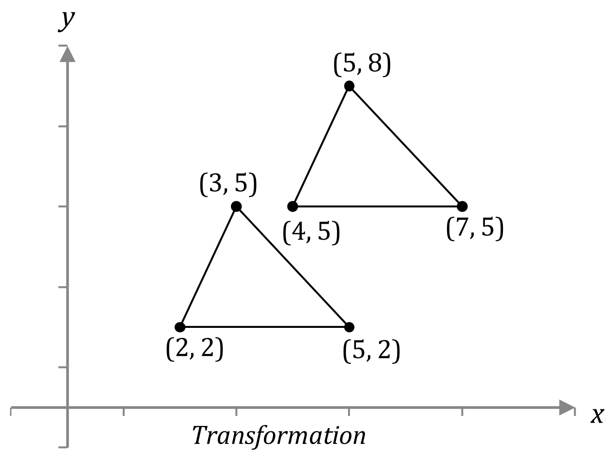
Use the distance formula to find the length of one of the sides:

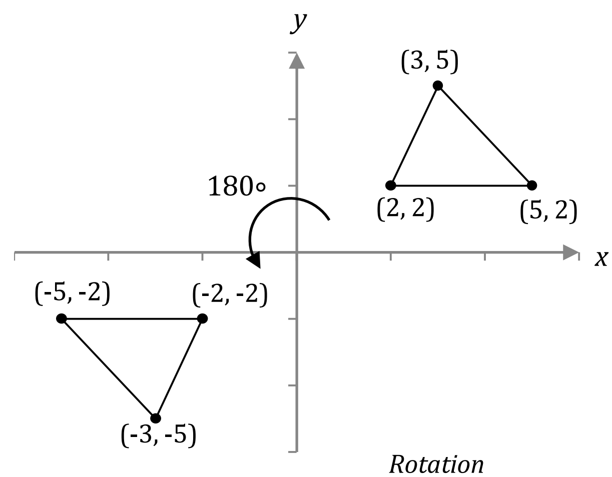
$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
$$d = \sqrt{(3 - 0)^2 + (0 - 3)^2}$$
$$d = \sqrt{18}$$

The area of the square is $(\sqrt{18})^2 = 18$ units squared.

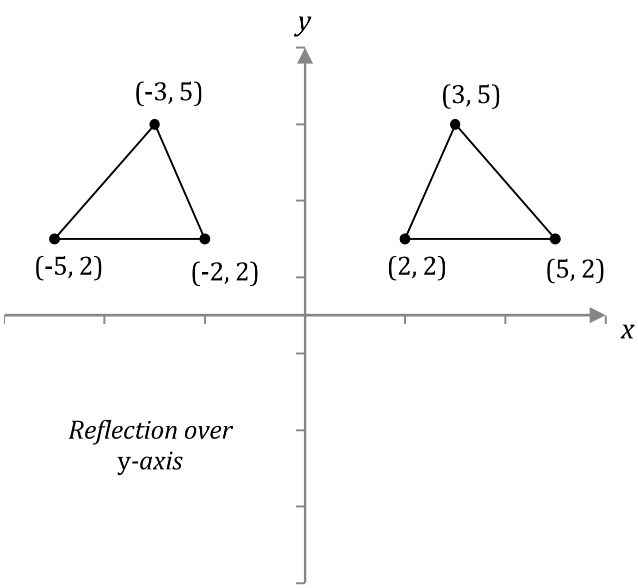
A **transformation** moves an object horizontally and/or vertically along the coordinate plane without changing its size or shape. If a polygon is transformed 2 positive units horizontally and 3 positive units vertically, then for each of its vertices the x-coordinate will increase by 2 units and the y-coordinate will increase by 3 units.

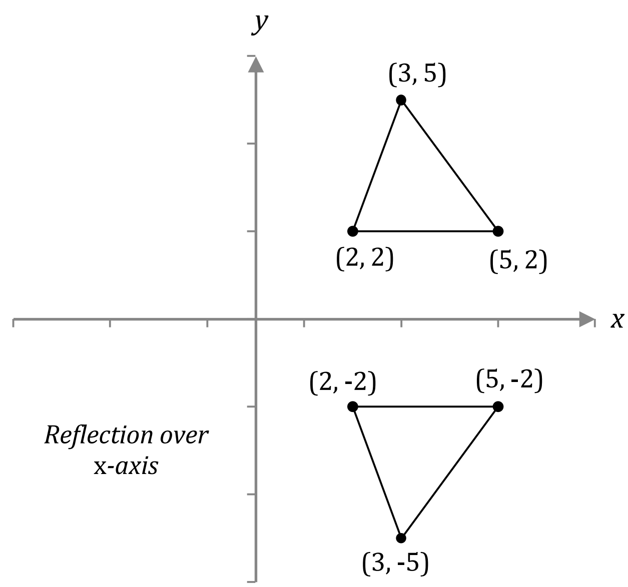
A **rotation** turns an object around a point, called the **centre of rotation**. If a point with coordinates (x, y) is rotated 180° around the origin, its resulting coordinates will be $(-x, -y)$.





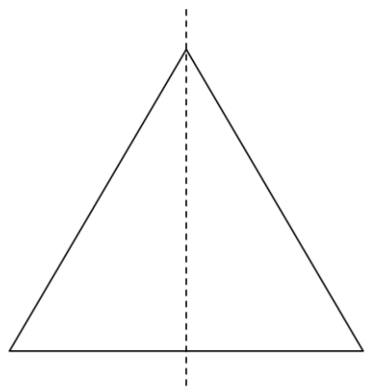
A **reflection** creates a mirror image of an object over a line, called the **line of reflection**. A point and its reflection will be the same distance away from the line of reflection. If a point (x, y) is reflected about the x-axis, its resulting coordinates will be $(x, -y)$. If a point (x, y) is reflected about the y-axis, its resulting coordinates will be $(-x, y)$.



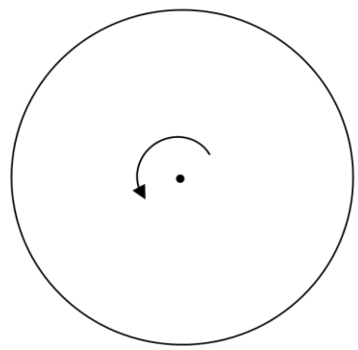


A figure has **reflectional symmetry** if there is a line that divides it into two halves, such that each half is the other half's perfect reflection about the line. In other words, if the figure were reflected about the **line of symmetry**, the result would be exactly the same figure. A figure may have multiple lines of symmetry.

A figure has **rotational symmetry** if there is a point around which a figure can be rotated a certain number of degrees, creating a resulting figure exactly the same as the original. This point is called the **point of symmetry**.

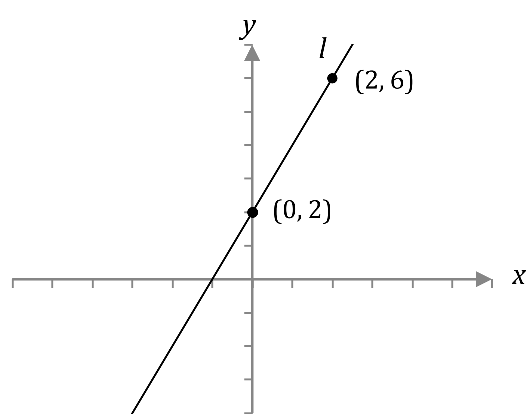


Reflectional symmetry



Rotational symmetry

- The figure to the right shows the graph of line l in the xy-coordinate plane. If line m is the reflection of line l in the x-axis, what is the equation of line m?

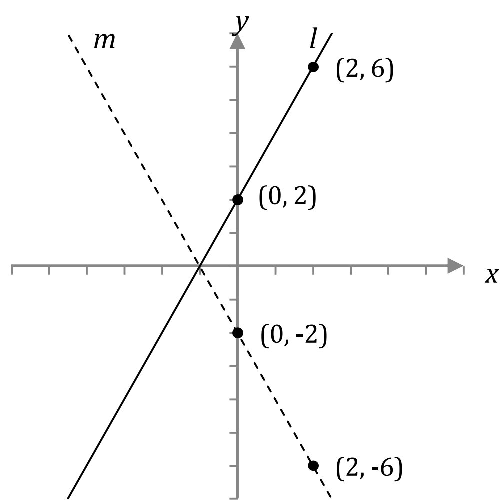
  

  If line m is the reflection of line l in the x-axis, then every point on line l will have a corresponding point on line m. These points will have the same x-coordinate but the negative y-coordinate:

  $$(0, -2)$$
  $$(2, -6)$$

  Use these points to write an equation for line m. The y-intercept is -2 and the slope is also -2, so the equation for line m is

  $$y = -2x - 2$$

IV. Data Analysis and Probability

- ***Counting***
- ***Combinations and permutations***
- ***Probability***
- ***Means, medians, and modes***

When **counting** how many integers are present between two endpoints, it is important to note whether the endpoints are being included.

- The number of integers between the endpoints n and m, inclusive, is $(n - m) + 1$.
- The number of integers between the endpoints n and m, exclusive, is $(n - m) - 1$.
- If only one of the enpoints is being included, then the number of integers from n to m is $(n - m)$.

- From 8:15pm to 9:20pm, the 1st through the 5th people in the waiting room were seen by a doctor. What was the average rate of patients seen per minute?

  Because the 1st *through* the 5th people were seen, our endpoints of 1 and 7 are being included. The number of people seen is $(5 - 1) + 1 = 5$.

  There are 65 minutes *from* 8:15pm *to* 9:20pm; only one of these endpoints is being included. Thus, the rate of patients seen per minute is $\frac{5}{65} = \frac{1}{13}$.

The **counting principle** tells us that, for two tasks, if there are m ways to complete the first and n ways to complete the second, then there are $m \times n$ ways to complete the two of them. Most of these questions involve **permutations**, where the order of the two tasks is important.

- To determine the 1st, 2nd, and 3rd prize of a lottery drawing, three tickets are selected one at a time from a pool of 50 total tickets, with no repeats allowed. How many different combinations of 1st, 2nd, and 3rd-prize tickets are possible?

  There are 50 possible tickets for the first drawing. For the second, only 49 are left, and for the third, there are only 48 possibilities. Thus, the number of different combinations (actually, permutations) is $50 \times 49 \times 48 = 117600$.

- John wants to paint his bedroom, bathroom, and kitchen either pink, yellow, blue, or green. If any rooms can be the same colour, how many combinations of colours are possible?

For the first room, John has 4 possible colours. Because any rooms can be the same colour, he still has 4 possibilities for the second room and 4 possibilities for the third. There are thus $4 \times 4 \times 4 = 64$ permutations of colours possible.

In some questions, the order of tasks is not important. For instance, if Gary, Mark, and Diana are being divided into pairs, the pair Gary-Mark is the same as the pair Mark-Gary—it doesn't matter which one is chosen first. These are called **combinations** as opposed to permutations, though the SAT does not use this terminology.

- Sarah wants to wear two bracelets to school tomorrow. If she has five bracelets to choose from and it does not matter in what order she wears them, how many combinations of bracelets are possible?

  Sarah has five options for her first bracelet and four options for her second. This would give $5 \times 4 = 20$ permutations possible. However, it doesn't matter what order she chooses the bracelets: a pink-yellow bracelet combination would be the same as a yellow-pink bracelet combination. Thus, each of our combinations is erroneously being counted two times. The actual number of combinations possible is $\frac{20}{2} = 10$.

The **probability** that an event will take place is expressed mathematically as a number from 0 to 1. A probability of 0 means that the event is impossible, and a probability of 1 means that an event is completely certain. For everything in between, probability is represented as a ratio:

$$\frac{number\ of\ favorable\ outcomes}{number\ of\ possible\ outcomes}$$

- An integer between 0 and 99, inclusive, is chosen at random. What is the probability that the integer will end in 0?

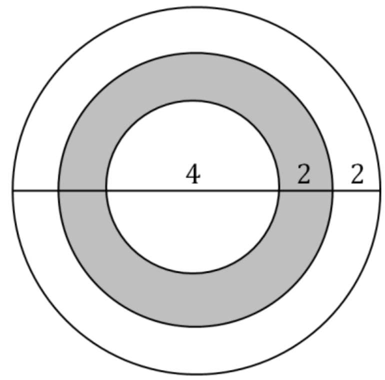
  There are 10 integers between 0 and 99 that end in 0: 0, 10, 20, 30, 40, 50, 60, 70, 80, and 90. The number of favorable outcomes is 10. The number of possible outcomes is $(90 - 0) + 1 = 100$, as there are 100 integers total to choose from. Thus, the probability that the integer chosen will end in 0 is

$$\frac{number\ of\ favorable\ outcomes}{number\ of\ possible\ outcomes} = \frac{10}{100} = \frac{1}{10}$$

Geometric probability questions ask you to find the probability that an event will occur within a specific region of a geometric figure. In these cases, the "number of favorable outcomes" is the area of the region in question, and the "number of possible outcomes" is the area of the whole figure.

$$\frac{area\ of\ specific\ region}{area\ of\ whole\ figure}$$

- The figure to the right shows a target made of concentric circles. If the diameter of the inner circle is 4 inches and each of the circles are spaced 2 inches apart, what is the probability that a dart thrown at random will land in the shaded region?



  We need to calculate the area of the shaded region and the area of the entire figure. The radius of the inner circle is $\frac{4}{2}$ = 2 inches, so its area is $\pi(2^2)$ = 4π inches squared. The radius of the next circle is 2 inches more, so its area is $\pi(4^2)$ = 16π inches squared. Finally, the radius of the largest circle is another 2 inches more, so its area is $\pi(6^2)$ = 36π inches squared.

  The area of the entire figure is 36π inches squared. The area of the shaded region is the area of the second circle minus the area of the inner circle: $16\pi - 4\pi = 12\pi$ inches squared. Thus, the probability that a dart will land in the shaded region is

$$\frac{area\ of\ specific\ region}{area\ of\ whole\ figure} = \frac{12\pi}{36\pi} = \frac{1}{3}$$

Two events are **independent** of each other if the probability of one event occurring does not affect in any way the probability of the other event occurring. For instance, if you were to roll two dice, you would have a $\frac{1}{6}$ probability of rolling a 1 for each die. For independent events like these, you can multiply the individual probabilities together to find the probability of both events occurring at the same time. Thus, the probability that you will roll a 1 on both dice is $\frac{1}{6} \times \frac{1}{6} = \frac{1}{36}$.

- Jesse has two pairs of grey socks and one pair of blue socks. If he dresses himself in the dark and selects two socks randomly from the drawer, one at a time, what is the probability that he will pick two grey socks?

  Jesse has 6 socks total in his drawer, of which 4 are grey. The probability that he will select one grey sock is $\frac{4}{6} = \frac{2}{3}$. He then has only 5 socks in his drawer, of which 3 are grey, so the probability that he will select another grey sock is $\frac{3}{5}$. The probability that he select two grey socks is $\frac{2}{3} \times \frac{3}{5} = \frac{6}{15}$.

An **average** is the same as an arithmetic **mean**. If your data comprises n numbers, then the average of those n numbers is their sum divided by n:

$$average = \frac{sum}{n}$$

- After her third English test, Amanda's average test score was 88. What score does she need on her fourth test in order to have an average of 90?

  Her current average is the sum of her first three test scores divided by three. We can't figure out what each individual score was, but we can find their sum:

$$average = \frac{sum}{n}$$
$$88 = \frac{sum}{3}$$
$$sum = 264$$

  For her average after her fourth test to equal 90, the sum of all four test scores divided by four must equal 90. If the sum of her first three test scores is 264, let x equal the fourth test score:

$$90 = \frac{264 + x}{4}$$
$$360 = 264 + x$$
$$x = 96$$

  Amanda needs a score of 96 on her fourth test.

- What is the average of $8x + 4$ and $-2x$?

  Treat these two expressions like any other numbers: add them together and then divide by two.

$$average = \frac{(8x + 4) + (-2x)}{2} = \frac{6x + 4}{2} = 3x + 2$$

A **weighted average** is the average of several groups of data, each with an unequal size (weight). To find the average, each number must be multiplied by its weighting factor to reflect how many times it appears in the total data set. These products are then added together and divided by the total number of numbers in the data set.

- At a fundraiser, 25 people donated an average of \$50 each and 13 people donated an average of \$100 each. What was the average donation for the entire fundraiser?

We must weight each donated amount according to the number of people who donated it, then divide by the total number of people at the event:

$$average = \frac{25(\$50) + 13(\$100)}{38} = \frac{2550}{38} = \$67.11$$

The average donation was $67.11.

To find the **median** of a group of data, put the data into numerical order and look for the middle value. If there is no middle value, the median is the average of the two middle numbers. The **mode** is the value that occurs most frequently; a data set can have multiple modes.

- The most recent exam scores for a class were 64, 83, 97, 95, 83, 72, 75, 78, 78, and 87. What was the average score? What was the median score? What is/are the mode(s) of this data?

  Put the data first into numerical order: 64, 72, 75, 78, 78, 83, 83, 87, 95, 98

  The average of the data is

  $$\frac{64 + 72 + 75 + 78 + 78 + 83 + 83 + 87 + 95 + 98}{10} = 81.2$$

  The median score lies evenly between 78 and 83:

  $$\frac{78 + 83}{2} = 80.5$$

  The modes are 78 and 83.